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Original Research Paper

Bode Diagram and Mathematical Functions with MAXIMA

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The methods of calculus lie at the heart of the physical sciences and engineering. Maxima can help you make faster progress, if you are just learning calculus. The examples in this research paper will offer an opportunity to see some Maxima tools in the context of simple examples, but you will likely be thinking about much harder problems you want to solve as you see these tools used here. This research paper includes Bode Diagram and Mathematical Functions with MAXIMA.

Keywords: Bode Diagram, Mathematical Functions, Trigonometric Functions, Hyperbolic Functions, Maxima.

1. INTRODUCTION

Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, sets, lists, vectors, matrices, tensors, and more. Maxima yields high precision numeric results by using exact fractions, arbitrary precision integers, and variable precision floating point numbers. Maxima can plot functions and data in two and three dimensions.

Maxima source code can be compiled on many computer operating systems, including Windows, Linux, and MacOS X. The source code for all systems and precompiled binaries for Windows and Linux are available at the SourceForge file manager.

Maxima is a descendant of Macsyma, the legendary computer algebra system developed in the late 1960s at the Massachusetts Institute of Technology. It is the only system based on that effort still publicly available and with an active user community, thanks to its open source nature. Macsyma

was revolutionary in its day, and many later systems, such as Maple and Mathematica, were inspired by it.

The Maxima branch of Macsyma was maintained by William Schelter from 1982 until he passed away in 2001. In 1998 he obtained permission to release the source code under the GNU General Public License (GPL). It was his efforts and skills that made the survival of Maxima possible.

MAXIMA has been constantly updated and used by researcher and engineers as well as by students.

2. BODE DIAGRAM

A Bode plot is a graph of the transfer function of a linear, time-invariant system versus frequency, plotted with a log-frequency axis, to show the system's frequency response. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response gain, and a Bode phase plot, expressing the frequency response phase shift. Maxima uses for Bode diagram the package "bode" and the following functions.

Table 1:

Function	Description
<code>bode_gain (H, range, ...plot_opts...)</code>	Function to draw Bode gain plots.
<code>bode_phase (H, range, ...plot_opts...)</code>	Function to draw Bode phase plots.

2.1. Examples

Example 2.1.1.

Draw the gain and phase for the following functions in frequency domain:

- a) $H(s)=1/(1+s)$
- b) $H(s)=1/(1+s+s^2)$
- c) $H(s)=(1+s)/(1+s+s^2)$

Solutions:

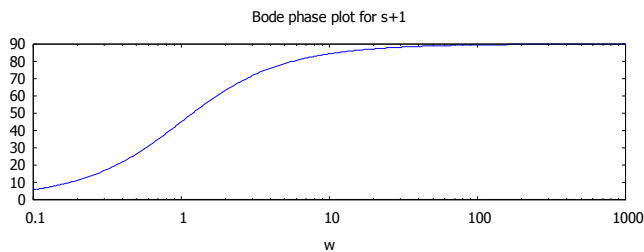
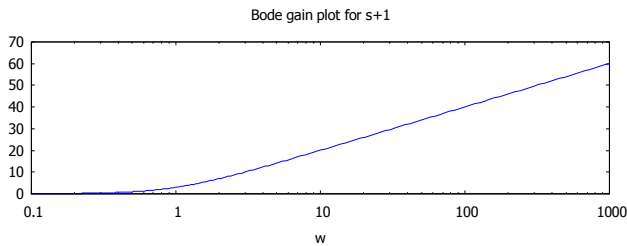
a)

```
(%i1) load(bode)$
H1(s):=1/1+s$

bode_gain(H1(s), [w,1/10,1000]);

bode_phase(H1(s), [w,1/10,1000]);

(%o3)
(%o4)
```



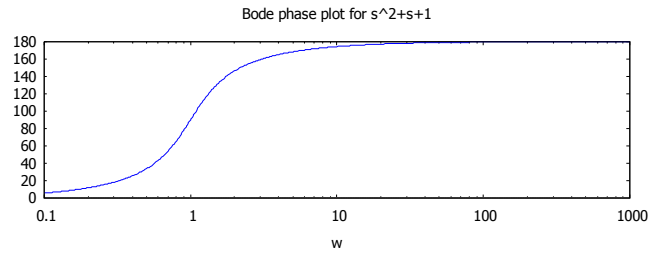
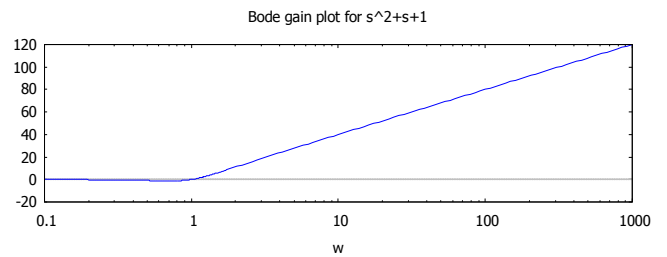
b)

```
(%i5) load(bode)$
H1(s):=1/1+s+s^2$

bode_gain(H1(s), [w,1/10,1000]);

bode_phase(H1(s), [w,1/10,1000]);

(%o7)
```



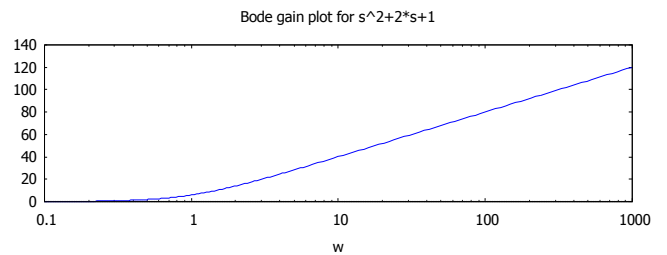
c)

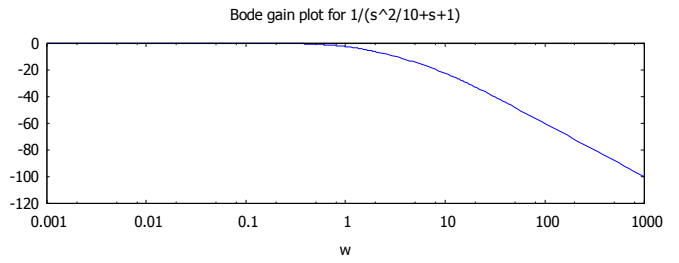
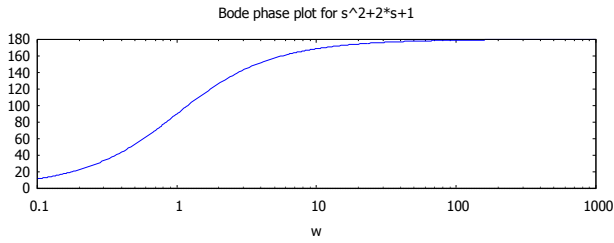
```
(%i9) load(bode)$
H1(s):=(1+s)/1+s+s^2$

bode_gain(H1(s), [w,1/10,1000]);

bode_phase(H1(s), [w,1/10,1000]);

(%o11)
(%o12)
```





Example 2.1.2.

Draw the gain and phase for the following functions in frequency domain for the initial frequency $\omega_0 = 10$.

- a) $H(s) = 1/(1+s)$
- b) $H(s) = 1/(1+s+s^2)$
- c) $H(s) = (1+s)/(1+s+s^2)$

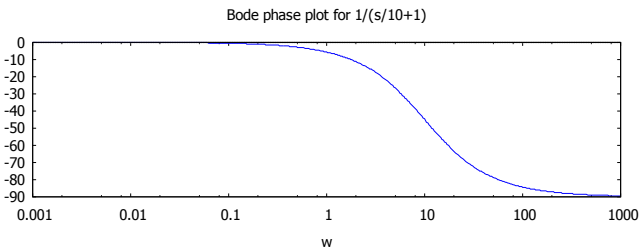
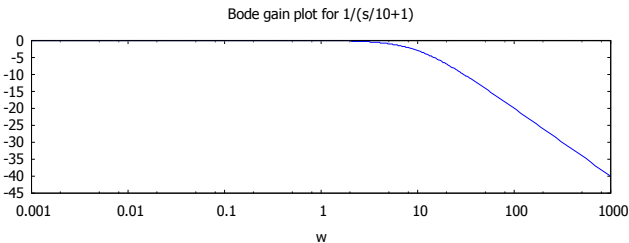
Solutions:

a)

```
(%i13) load(bode) $
      H2(s) := 1/(1+s/omega0) $

      bode_gain(H2(s), [w, 1/1000, 1000]), omega0=10;

      bode_phase(H2(s), [w, 1/1000, 1000]), omega0=10;
(%o15)
(%o16)
```



b)

```
(%i17) load(bode) $
      H2(s) := 1/(1+s+s^2/omega0) $

      bode_gain(H2(s), [w, 1/1000, 1000]), omega0=10;

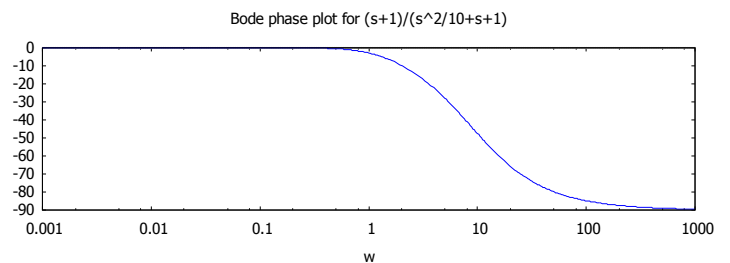
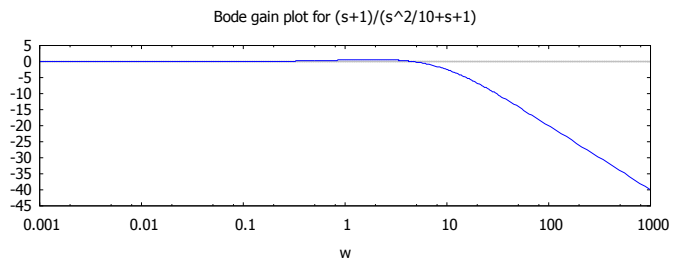
      bode_phase(H2(s), [w, 1/1000, 1000]), omega0=10;
(%o19)
(%o20)
```

c)

```
(%i21) load(bode) $
      H2(s) := (1+s)/(1+s+s^2/omega0) $

      bode_gain(H2(s), [w, 1/1000, 1000]), omega0=10;

      bode_phase(H2(s), [w, 1/1000, 1000]), omega0=10;
(%o23)
(%o24)
```



3.0. MATHEMATICAL FUNCTIONS

Table 1: In the following table some selected mathematical functions are given.

Function	Description
abs (<i>expr</i>)	Returns the absolute value <i>expr</i> . If <i>expr</i> is complex, returns the complex modulus of <i>expr</i> .
cabs (<i>expr</i>)	Returns the complex absolute value (the complex modulus) of <i>expr</i> .
conjugate (<i>x</i>)	Returns the complex conjugate of <i>x</i> .
imagpart (<i>expr</i>)	Returns the imaginary part of the expression <i>expr</i> . imagpart is a computational function, not a simplifying function.
polarform (<i>expr</i>)	Returns an expression $r e^{i\theta}$ equivalent to <i>expr</i> , such that <i>r</i> and <i>theta</i> are purely real.
realpart (<i>expr</i>)	Returns the real part of <i>expr</i> . realpart and imagpart will work on expressions involving trigonometric and hyperbolic functions, as well as square root, logarithm, and exponentiation.
rectform (<i>expr</i>)	Returns an expression $a + b i$ equivalent to <i>expr</i> , such that <i>a</i> and <i>b</i> are purely real.
binomial (<i>x</i> , <i>y</i>)	The binomial coefficient $x!/(y!(x-y)!)$. If <i>x</i> and <i>y</i> are integers, then the numerical value of the binomial coefficient is computed. If <i>y</i> , or <i>x - y</i> , is an integer, the binomial coefficient is expressed as a polynomial.
factorial Operator: !	Represents the factorial function. Maxima treats factorial (<i>x</i>) the same as $x!$. For any complex number <i>x</i> , except for negative integers, $x!$ is defined as $\Gamma(x+1)$.
exp (<i>x</i>)	Represents the exponential function. Instances of exp (<i>x</i>) in input are simplified to e^x ; exp does not appear in simplified expressions.
log (<i>x</i>)	Represents the natural (base <i>e</i>) logarithm of <i>x</i> .
sqrt (<i>x</i>)	The square root of <i>x</i> . It is represented internally by $x^{1/2}$
Trigonometric functions:	
	acos, acosh, acot, acoth, acsc, acsch, ... cos, cosh, ..sin, sinh,....
trigexpand (<i>expr</i>)	Expands trigonometric and hyperbolic functions of sums of angles and of multiple angles occurring in <i>expr</i> . For best results, <i>expr</i> should be expanded. To enhance user control of simplification, this function expands only one level at a time, expanding sums of angles or multiple angles.
trigreduce (<i>expr</i> , <i>x</i>) trigreduce (<i>expr</i>)	Combines products and powers of trigonometric and hyperbolic sin's and cos's of <i>x</i> into those of multiples of <i>x</i> . It also tries to eliminate these functions when they occur in denominators. If <i>x</i> is omitted then all variables in <i>expr</i> are used.
trigsimp (<i>expr</i>)	Employs the identities $\sin(x)^2 + \cos(x)^2 = 1$ and $\cosh(x)^2 - \sinh(x)^2 = 1$ to simplify expressions containing tan, sec, etc., to sin, cos, sinh, cosh.
trigrat (<i>expr</i>)	Gives a canonical simplified quasilinear form of a trigonometrical expression; <i>expr</i> is a rational fraction of several sin, cos or tan, the arguments of them are linear forms in some variables (or kernels) and π/n (<i>n</i> integer) with integer coefficients. The result is a simplified fraction with numerator and denominator linear in sin and cos. Thus trigrat linearize always when it is possible.
make_random_state (<i>n</i>) make_random_state (<i>s</i>) make_random_state (<i>true</i>) make_random_state (<i>false</i>)	A random state object represents the state of the random number generator. The state comprises 627 32-bit words. make_random_state (<i>n</i>) returns a new random state object created from an integer seed value equal to <i>n</i> modulo 2^{32} . <i>n</i> may be negative. make_random_state (<i>s</i>) returns a copy of the random state <i>s</i> . make_random_state (<i>true</i>) returns a new random state object, using the current computer clock time as the seed. make_random_state (<i>false</i>) returns a copy of the current state of the random number generator.

3.1. Examples

Example 3.1.1.

```
(%i1) eq:cos(4*x)*cos(x);
      trigexpand(eq);
(%o1) cos(x)cos(4 x)
      (%o2) cos(x)(sin(x)^4-6 cos(x)^2 sin(x)^2+cos(x)^4)
```

Example 3.1.2.

```
(%i3) eq:sin(x)^2+cos(x)^3;
      trigreduce(eq);
(%o3) sin(x)^2+cos(x)^3
      (%o4)  $\frac{\cos(3x)+3\cos(x)}{4} + \frac{1-\cos(2x)}{2}$ 
```

Example 3.1.3.

```
(%i9) eq:sin(x)^2-cos(x)^3;
      trigrat(eq);
(%o9) sin(x)^2-cos(x)^3
      (%o10)  $\frac{\cos(3x)+2\cos(2x)+3\cos(x)-4}{4}$ 
```

Example 3.1.4.

```
(%i11) eq:sin(x)^2-cos(x)^2;
      sqrt(eq);
(%o11) sin(x)^2-cos(x)^2
      (%o12)  $\sqrt{\sin(x)^2-\cos(x)^2}$ 
```

Example 3.1.5.

```
(%i13) eq:sin(x)^2-cos(x)^2;
      log(eq);
(%o13) sin(x)^2-cos(x)^2
      (%o14)  $\log(\sin(x)^2-\cos(x)^2)$ 
```

Example 3.1.6.

```
(%i15) eq:sin(x)^4;
      factorial(eq);
(%o15) sin(x)^4
      (%o16)  $(\sin(x)^4)!$ 
```

Example 3.1.7.

```
(%i17) eq:sin(x)^4;
      exp(eq);
(%o17) sin(x)^4
      (%o18) %esin(x)^4
```

4. CONCLUSION

The research paper can apply each and every part of Bode Diagram and Mathematical Functions, Bode Diagram and Mathematical Functions help application of the physical sciences and engineering, make faster progress, and help to understand Laplace Transforms, Fast Fourier Transform, Fourier Series and Taylor Series. The paper particularity helps to understand parts of Calculus and is going to extend to other parts of the Calculus.

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