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Original Research Article

Algebra of Matrices and Determinants with MAXIMA Entrance

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The methods of calculus lie at the heart of the physical sciences and engineering. Maxima can help you make faster progress, if you are just learning calculus. The examples in this research paper will offer an opportunity to see some Maxima tools in the context of simple examples, but you will likely be thinking about much harder problems you want to solve as you see these tools used here. This research paper includes linear algebra with MAXIMA.

Keywords: Algebra, Matrices, Determinants and Maxima.

1. INTRODUCTION

Maxima is a system for the manipulation of symbolic and numerical expressions, including differentiation, integration, Taylor series, Laplace transforms, ordinary differential equations, systems of linear equations, polynomials, sets, lists, vectors, matrices, tensors, and more. Maxima yields high precision numeric results by using exact fractions, arbitrary precision integers, and variable precision floating point numbers. Maxima can plot functions and data in two and three dimensions.

Maxima source code can be compiled on many computer operating systems, including Windows, Linux, and MacOS X. The source code for all systems and precompiled binaries for Windows and Linux are available at the SourceForge file manager.

Maxima is a descendant of Macsyma, the legendary computer algebra system developed in the late 1960s at the Massachusetts Institute of Technology. It is the only system based on that effort still publicly available and with an active user community, thanks to its open source nature. Macsyma was revolutionary in its day, and many later systems, such as Maple and Mathematica, were inspired by it. The Maxima branch of Macsyma was maintained by William Schelter from 1982 until he passed away in 2001. In 1998 he obtained permission to release the source code under the GNU General Public License (GPL). It was his efforts and skills that made the survival of Maxima possible.

MAXIMA has been constantly updated and used by researcher and engineers as well as by students

2. ALGEBRA OF MATRICES

2.1. Classical Method

2.1.1. Introduction

A matrix A is a rectangular array of scalars usually presented in the following form:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \quad (2.1)$$

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The rows of the matrix **A** are m horizontal lists of scalars, and the columns are n vertical lists of scalars.
A square matrix is a matrix with the same number of rows as columns. An n _ n square matrix is said to be of order n (n-square matrix):

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \quad (2.2)$$

A diagonal matrix consists of the elements with the same subscripts:

$$\begin{pmatrix} a_{11} & \cdots & \\ \vdots & a_{22}\ddots & \vdots \\ & \cdots & a_{nn} \end{pmatrix} \quad (2.3)$$

The tace of A is the sum of the diagonal elements:

$$tr(\mathbf{A}) = a_{11} + a_{22} + a_{33} + ...a_{nn} \quad (2.4)$$

A unit matrix **I** is a n-square matrix with 1's on the diagonal and 0's elsewhere.

$$\begin{pmatrix} 1 & & 0 \\ & 1\ddots & \\ 0 & & 1 \end{pmatrix}_{nn} \quad (2.5)$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ 0\cdots & a_{22}\ddots & \vdots \\ 0\cdots & \cdots & a_{nn} \end{pmatrix} \quad (2.6)$$

A square matrix is upper triangular if all entries below the main diagonal are equal to 0:

A matrix is symmetric if symmetric elements, mirror elements with respect to the diagonal are equal,

$$a_{ij} = a_{ji} \quad (2.7)$$

The inverse of a square matrix **A**, sometimes called a reciprocal matrix, is a matrix $\mathbf{A}^{-1}$ such that

$$\mathbf{A} * \mathbf{A}^{-1} = \mathbf{I} \quad (2.8),$$

where **A** is the identity matrix. $\mathbf{A}^T$ created by any one of the following equivalent actions:

- reflect A over its main diagonal (which runs top-left to bottom-right) to obtain $\mathbf{A}^T$
- write the rows of **A** as the columns of $\mathbf{A}^T$
- write the columns of **A** as the rows of $\mathbf{A}^T$

Formally, the ith row, jth column element of $\mathbf{A}^T$ is the jth row, ith column element of **A**:

$$\left[\mathbf{A}^T\right]_{ij} = \left[\mathbf{A}\right]_{ji} \quad (2.9)$$

If A is an m * n matrix then $\mathbf{A}^T$ is an n * m matrix.

$$\mathbf{A}^T = \begin{pmatrix} a_{11} & a_{21}\cdots & a_{m1} \\ a_{12} & a_{22}\ddots & \vdots \\ a_{1n} & \cdots & a_{nn} \end{pmatrix} \quad (2.10)$$

2.1.2. Matrix Addition

Consider the two matrices $\mathbf{A} = \left[a_{ij}\right] \quad and \quad \mathbf{B} = \left[b_{ij}\right]$ with the same size m*n matrices. The sum of these matrices is obtained by adding corresponding elements from these matrices.

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12}\cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22}\ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2}\cdots & a_{mn}+b_{mn} \end{pmatrix} \quad (2.11)$$

2.1.2.1. Examples

Example 2.1.

Given two matrices as

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}.$$ Then

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

Example 2.2.

Given two matrices as

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix}.$$ Then

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1+4 & 2+5 \\ 3+6 & 4+7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 11 \end{pmatrix}$$ and

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1-4 & 2-5 \\ 3-6 & 4-7 \end{pmatrix} = \begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}$$

2.1.3. Scalar Multiplication

The product of the matrix $\mathbf{A}=\left[a_{ij}\right]$ by a scalar k is the matrix obtained by multiplying each element of $\mathbf{A}=\left[a_{ij}\right]$ by k.

$$k\mathbf{A}=\begin{pmatrix} ka_{11} & ka_{12}\cdots & ka_{1n} \\ ka_{21} & ka_{22}\ddots & \vdots \\ ka_{m1} & ka_{m2}\cdots & ka_{mn} \end{pmatrix} \quad (2.12)$$

2.1.3.1. Examples

Example 2.3.

Given the matrix **A** as

$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and k=c. Then $k\mathbf{A}=\begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$.

Example 2.4.

Given the matrix A as

$\mathbf{A}=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and k=2. Then $k\mathbf{A}=\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$.

2.1.4. Matrix Multiplication

Consider the matrices $\mathbf{A}=\left[a_{ik}\right] \quad and \quad \mathbf{B}=\left[b_{kj}\right]$ which are matrices such that the number of columns of **A** is equal to the number of rows of **B**.

Is **A** an m*n matrix and **B** is a p*n matrix, then the product **AB** is the m*n matrix whose ij-entry is obtained by multiplying the ith row of **A** by the jth column of **B**.

$$\mathbf{C = AxB} = \begin{pmatrix} a_{11} & \cdots & a_{ip} \\ \cdots & & \\ a_{i1} & \ddots & a_{ip} \\ \vdots & & \\ a_{m1} & \cdots & a_{mp} \end{pmatrix} x \begin{pmatrix} b_{11} & \cdots b_{1j}\cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{p1} & b_{pj} & b_{pm} \end{pmatrix}$$

$$\mathbf{C = AxB} = \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & c_{ij}\ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{pmatrix} \quad (2.13)$$

where

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+...+a_{ip}b_{pj}=\sum_{k=1}^{p}a_{ik}b_{kj} \quad (2.14)$$

2.1.4.1. Examples

Example 2.5.

Given the matrix **A** as

$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad and \quad \mathbf{B}=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$. Then

$$\mathbf{A}x\mathbf{B}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} x \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$\mathbf{A}x\mathbf{B}=\begin{pmatrix} (a_{11}b_{11}+a_{12}b_{21}) & (a_{11}b_{12}+a_{12}b_{22}) \\ (a_{21}b_{11}+a_{22}b_{21}) & (a_{21}b_{12}+a_{22}b_{22}) \end{pmatrix}.$$

Example 2.6.

Given the matrix **A** as

$\mathbf{A}=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} \quad and \quad \mathbf{B}=\begin{pmatrix} 8 & 7 \\ 6 & 5 \end{pmatrix}$. Then

$$\mathbf{A}x\mathbf{B}=\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} x \begin{pmatrix} 8 & 7 \\ 6 & 5 \end{pmatrix},$$

$$\mathbf{C}=\mathbf{A}x\mathbf{B}=\begin{pmatrix} (4x8+3x6) & (4x7+3x5) \\ (2x8+2x6) & (2x7+1x5) \end{pmatrix}=\begin{pmatrix} 50 & 43 \\ 28 & 19 \end{pmatrix}.$$

2.2. MAXIMA APPLICATIONS

2.2.1. Introduction

2.2.2. Matrix Addition

```
(%i1) A:matrix([a11,a12],[a21,a22]);
(%o1) [a11 a12]
      [a21 a22]
(%i2) B:matrix([b11,b12],[b21,b22]);
(%o2) [b11 b12]
      [b21 b22]
(%i3) A+B;
(%o3) [b11+a11 b12+a12]
      [b21+a21 b22+a22]
(%i4) A-B;
(%o4) [a11-b11 a12-b12]
      [a21-b21 a22-b22]
```

2.2.3. Scalar Multiplication

```
(%i5) A:matrix([a11,a12],[a21,a22]);
(%o5) [a11 a12]
      [a21 a22]
```

Table 1: Some selected matrix functions from MAXIMA are given in the following table:

Function	Description
entermatrix(m,n)	Returns an m by n matrix reading the elements
genmatrix(a)	Returns a matrix generated from a
matrix(row1,rown)	Returns a rectangular matrix
invert(M)	Returns the inverse of matrix **M**
transpose(M)l	Returns the transpose of matrix **M**
triangularize(M)	Returns the upper triangulare of matrix **M**
zeromatrix(m,n)	Returns an m by n matrix, all elements of zero
addcol(A,B)	Matrices can be stacked
addrowl(A,B)	Matrices can be stacked
row(A,i)	Returns the ith row
col(A,j)	Returns the jth column

(%i6) k*A;

(%o6) $\begin{bmatrix} a11\,k & a12\,k \\ a21\,k & a22\,k \end{bmatrix}$

2.2.4. **Matrix Multiplication**

(%i7) A:matrix([a11,a12],[a21,a22]);

(%o7) $\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$

(%i8) B:matrix([b11,b12],[b21,b22]);

(%o8) $\begin{bmatrix} b11 & b12 \\ b21 & b22 \end{bmatrix}$

(%i9) A.B;

(%o9) $\begin{bmatrix} a12\,b21+a11\,b11 & a12\,b22+a11\,b12 \\ a22\,b21+a21\,b11 & a22\,b22+a21\,b12 \end{bmatrix}$

(%i10) B.A;

(%o10) $\begin{bmatrix} a21\,b12+a11\,b11 & a22\,b12+a12\,b11 \\ a21\,b22+a11\,b21 & a22\,b22+a12\,b21 \end{bmatrix}$

2.2.5. **Numerical Example**

Example 1.

(%i11) A:matrix([1,-1,2],[2,-2,3],[3,-3,4]);

(%o11) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix}$

(%i12) B:matrix([-1,-1,2],[-2,-2,3],[-3,-3,4]);

(%o12) $\begin{bmatrix} -1 & -1 & 2 \\ -2 & -2 & 3 \\ -3 & -3 & 4 \end{bmatrix}$

(%i13) A.B;

(%o13) $\begin{bmatrix} -5 & -5 & 7 \\ -7 & -7 & 10 \\ -9 & -9 & 13 \end{bmatrix}$

(%i14) B.A;

(%o14) $\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 2 \\ 3 & -3 & 1 \end{bmatrix}$

Example 2

(%i15) A:matrix([p,k],[l,m]);

(%o15) $\begin{bmatrix} p & k \\ l & m \end{bmatrix}$

(%i16) B:addcol(A,A);

(%o16) $\begin{bmatrix} p & k & p & k \\ l & m & l & m \end{bmatrix}$

(%i17) B:addrow(A,A);

(%o17) $\begin{bmatrix} p & k \\ l & m \\ p & k \\ l & m \end{bmatrix}$

Example 3.

(%i18) A:matrix([1,-1,z],[2,y,3],[x,-3,4]);

(%o18) $\begin{bmatrix} 1 & -1 & z \\ 2 & y & 3 \\ x & -3 & 4 \end{bmatrix}$

(%i19) row(A,1);

(%o19) $\begin{bmatrix} 1 & -1 & z \end{bmatrix}$

(%i20) col(A,3);

(%o20) $\begin{bmatrix} z \\ 3 \\ 4 \end{bmatrix}$

Example 4.

(%i21) A:matrix([1,-1,z],[2,y,3],[x,-3,4]);

(%o21) $\begin{bmatrix} 1 & -1 & z \\ 2 & y & 3 \\ x & -3 & 4 \end{bmatrix}$

```
(%i22) invert(A);
```

$$(\%o22)\quad \begin{bmatrix} \frac{4y+9}{(-xy-6)z+4y-3x+17} & \frac{4-3z}{(-xy-6)z+4y-3x+17} & \frac{-yz-3}{(-xy-6)z+4y-3x+17} \\ \frac{3x-8}{(-xy-6)z+4y-3x+17} & \frac{4-xz}{(-xy-6)z+4y-3x+17} & \frac{2z-3}{(-xy-6)z+4y-3x+17} \\ \frac{-xy-6}{(-xy-6)z+4y-3x+17} & \frac{3-x}{(-xy-6)z+4y-3x+17} & \frac{y+2}{(-xy-6)z+4y-3x+17} \end{bmatrix}$$

Example 5.

```
(%i27) row1:[1,2,z]$
       row2:[1,y,2]$
       row3:[x,1,2]$
       mat:matrix(row1,row2,row3);
```

$$(\%o30)\quad \begin{bmatrix} 1 & 2 & z \\ 1 & y & 2 \\ x & 1 & 2 \end{bmatrix}$$

```
(%i31) row2:row1-row2$
       row3:row1-row3$
       mat:matrix(row1,row2,row3);
```

$$(\%o33)\quad \begin{bmatrix} 1 & 2 & z \\ 0 & 2-y & z-2 \\ 1-x & 1 & z-2 \end{bmatrix}$$

3. DETERMİNANTS

3.1. Classical Methd

3.1.1. Definitions

The determinant det A of an nxn matrix A, for $n \geq 2$, is defined as the **Laplace expansion** of the determinant, using submatrices of A.

For any 2 x 2 matrix

$$\mathbf{A} = \left[a_{ij}\right] = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (3.1)$$

is the det **A** defined as:

$$\det \mathbf{A} = a_{11}a_{22} - a_{12}a_{21} \quad (3.2)$$

For any 3x3 matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (3.3)$$

is the det **A** defined as:

$$\det \mathbf{A} = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} \quad (3.4)$$

The ijth minor $\mathbf{M}_{ij}$ of (n) x (n) matrix **A** is the (n - 1) x(n - 1) submatrix of **A** obtained by deleting the ith row and the jth column of **A**. Assuming that the determinants det $\mathbf{M}_{ij}$ of the minors of **A** are known, and use them to define the determinant of **A** itself.

The ijth **cofactor** $\mathbf{C}_{ij}$ of an n x n matrix **A** is $(-1)^{i+j}$ det $\mathbf{M}_{ij}$. The formula $(-1)^{i+j}$ assigns +1 or -1 to det $\mathbf{M}_{ij}$ depending on whether i + j is even or odd. Using the cofactors, the det of **A** is defined.

For any (n) x (n) matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \quad (3.5)$$

The determinant of **A** is defined as:

$$\det \mathbf{A} = |\mathbf{A}| = \sum_{\sigma} (\operatorname{sgn} \sigma) \mathrm{a}_{1 j_1} \mathrm{a}_{2 j_1} \ldots \mathrm{a}_{n j_n} \quad (3.6)$$

where $\sigma = j_1 j_2 \ldots j_n$

Example 3.1.

Consider the following matrix and calculate its determinant using cofactors:

$$\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

Solution:

By definition,

$$\det \mathbf{A} = a_{11}\mathbf{C}_{11} + a_{12}\mathbf{C}_{12} + a_{13}\mathbf{C}_{13} \quad (3.7)$$

The cofactors $\mathbf{C}_{ij}$ are:

$$\mathbf{C}_{11} = (-1)^{1+1} \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} = +(a_{22}a_{33} - a_{23}a_{32})$$

$$\mathbf{C}_{12} = (-1)^{1+2} \det \begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} = -(a_{21}a_{33} - a_{23}a_{31})$$

$$\mathbf{C}_{13} = (-1)^{1+3} \det \begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = +(a_{21}a_{32} - a_{22}a_{31})$$

Therefore,

$$\det \mathbf{A} = a_{11}\mathbf{C}_{11} + a_{12}\mathbf{C}_{12} + a_{13}\mathbf{C}_{13}$$

and

$$\det \mathbf{A} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$\det \mathbf{A} = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

3.1.2. Cramer's Rule

The Cramer's rule is a procedure for solving linear systems, using the determinants. Consider the linear system

$\mathbf{Ax} = \mathbf{b}$, where the **Ax** is the coefficient matrix, **x** is the unknown vector, and **b** is the column vector.

The Cramer's rule is defined for each

$$1 \le i \le n$$

by

$$x_i = \frac{\det \mathbf{A}(\mathrm{i/b})}{\det \mathbf{A}}, \quad (3.8)$$

where the numerator is the matrix obtained from **Ax** by replacing the ith column of **Ax** by **b**.

Example 3.2.

Given the following linear system

$$x - y + 3z = 10$$
$$-x + 2y + z = 5$$
$$x - y + z = 5$$

Its matrix form is

Ax = b

$$\begin{pmatrix} 1 & -1 & 3 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$

Solution:

$$\det \mathbf{A} = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\det \mathbf{A} = -2$$

$$\mathbf{A}(1/\mathrm{b}) = \begin{pmatrix} 10 & -1 & 3 \\ 5 & 2 & 1 \\ 5 & -1 & 1 \end{pmatrix}, \quad \mathbf{A}(2/\mathrm{b}) = \begin{pmatrix} 1 & 10 & 3 \\ -1 & 5 & 1 \\ 1 & 5 & 1 \end{pmatrix},$$

$$\det(\mathbf{A}(1/\mathrm{b})) = -15 \qquad \det(\mathbf{A}(2/\mathrm{b})) = -10$$

$$\mathbf{A}(3/\mathrm{b}) = \begin{pmatrix} 1 & -1 & 10 \\ -1 & 2 & 5 \\ 1 & -1 & 5 \end{pmatrix}$$

$$\det(\mathbf{A}(3/\mathrm{b})) = -5$$

$$x = \frac{\det \mathbf{A}(1/\mathrm{b})}{\det \mathbf{A}} = \frac{-15}{-2} = 7.5, \quad y = \frac{\det \mathbf{A}(2/\mathrm{b})}{\det \mathbf{A}} = \frac{-10}{-2} = 5, \quad z = \frac{\det \mathbf{A}(3/\mathrm{b})}{\det \mathbf{A}} = \frac{-5}{-2} = 2.5$$

3.2. MAXIMA APPLICATIONS

3.2.1. Introduction

Determinant(**M**): returns the determinant of the matrix **M**

Example 3.3

```
(%i1) A:matrix([a11,a12],[a21,a22]);
(%o1) [a11 a12]
      [a21 a22]
(%i2) d:determinant(A);
(%o2) a11 a22-a12 a21
```

Example 3.4

```
(%i3) A:matrix([a11,a12,a13],[a21,a22,a23],[a31,a32,a33]);
(%o3) [a11 a12 a13]
      [a21 a22 a23]
      [a31 a32 a33]
(%i4) d:determinant(A);
(%o4) a11 (a22 a33-a23 a32) -a12 (a21 a33-a23 a31) +a13
(a21 a32-a22 a31)
```

Example 3.5

```
(%i5) A:matrix([2,3],[1,4]);
(%o5) [2 3]
      [1 4]
(%i6) d:determinant(A);
(%o6) 5
```

Example 3.6

```
(%i1) M:matrix([1,-1,3],[-1,2,1],[1,-1,1]);
(%o1) [ 1 -1 3]
      [-1  2 1]
      [ 1 -1 1]
(%i2) d:determinant(M);
(%o2) -2
(%i3) M1:matrix([10,-1,3],[5,2,1],[5,-1,1]);
(%o3) [10 -1 3]
      [ 5  2 1]
      [ 5 -1 1]
(%i4) d1:determinant(M1);
(%o4) -15
(%i5) M2:matrix([1,10,3],[-1,5,1],[1,5,1]);
(%o5) [ 1 10 3]
      [-1  5 1]
      [ 1  5 1]
(%i6) d2:determinant(M2);
(%o6) -10
(%i7) M3:matrix([1,-1,10],[-1,2,5],[1,-1,5]);
(%o7) [ 1 -1 10]
      [-1  2  5]
      [ 1 -1  5]
(%i8) d3:determinant(M3);
(%o8) -5
```

```
(%i10) x:d1/d;
(%o10) 15/2
(%i11) y:d2/d;
(%o11) 5
(%i12) z:d3/d;
(%o12) 5/2
```

4. CONCLUSION

The research paper can apply each and every part of Algebra of Matrices and Determinants, help application of the physical sciences and engineering, make faster progress, and help to understand Algebra of Matrices and Determinants faster. The paper particularity helps to understand parts of Linear Algebra and is going to extend to other parts of the Linear Algebra.

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